

# Are optically–selected QSO catalogs biased ?

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Received \_\_\_\_\_;    accepted \_\_\_\_\_

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## ABSTRACT

A thorough study of QSO–galaxy correlations has been done on a region close to the North Galactic Pole using a complete subsample of the optically selected CFHT/MMT QSO survey and the galaxy catalog of Odewahn and Aldering (1995). Although a positive correlation between bright QSOs and galaxies is expected because of the magnification bias effect, none is detected. On the contrary, there is a significant ( $> 99.6\%$ ) anticorrelation between  $z < 1.6$  QSOs and red galaxies on rather large angular distances. This anticorrelation is much less pronounced for high redshift  $z > 1.6$  QSOs, which seems to exclude dust as a cause of the QSO underdensity. This result suggests that the selection process employed in the CFHT/MMT QSO survey is losing up to 50% of low redshift  $z < 1.6$  QSOs in regions of high galaxy density. The incompleteness in the whole  $z < 1.6$  QSO sample may reach 10% and have important consequences in the estimation of QSO evolution and the QSO autocorrelation function.

*Subject headings:* cosmology: gravitational lensing — large-scale structure of universe, quasars: general, surveys

## 1. INTRODUCTION

There has been recently a flurry of papers searching for QSO–galaxy associations caused by gravitational lensing. The masses of foreground galaxies act as lenses on the light beams from background quasars. These QSOs will be affected in two ways: on one hand the beam will be focused, increasing the brightness of the source resulting in a higher population of quasars close (in angular distance) to foreground galaxies. On the other hand, the lensing will also magnify the solid angle, thereby decreasing the number density of QSOs. The tug between these two effects will determine the correlation between foreground and background sources. If the slope of the QSO number counts function is steep enough, there will be many faint quasars to allow an overdensity and thus a positive correlation, whereas a flat slope implies there are not many faint QSOs to balance the increase of solid angle, resulting in a negative correlation function (Narayan 1989; Schneider, Ehlers & Falco 1992). If the background sources are flux-limited in two uncorrelated wavebands ( e.g. radioloud QSOs) the total effect will be determined by the sum of the number counts slopes in both bands. This is called ‘double magnification bias’ (Borgeest, Von Linde & Refsdal 1991). So far, the observational results on large scales suggest a positive correlation for radio–selected QSO samples (Bartelmann & Schneider 1994 ; Benítez & Martínez-González 1995 ) and null correlation or even anticorrelation for radio quiet quasars. Although the magnification bias may produce anticorrelations for faint QSOs (as the ones in Boyle, Fong & Shanks 1988), they cannot explain the negative correlations found in Benítez & Martínez-González (1997) or Benítez, Ferreras & Martínez-González (1997) , where LBQS QSOs with ( $B_J \lesssim 18.7$ ) are considered. It is thus more plausible to interpret these anticorrelations as caused by dust extinction in foreground clusters (Romani & Maoz 1992) or selection biases due to the difficulty of identifying quasars in crowded fields (Maoz 1995).

It is interesting to confirm the existence of these biases and clarify their origin as

they may have important effects in the estimation of the QSO autocorrelation function or the QSO evolution. It is also remarkable that the conclusions obtained from gravitational lens surveys employing radioquiet quasars may be seriously affected by these selection effects. Along these lines we decided to explore the QSO-galaxy correlation function for the greyness-based CFHT/MMT survey. Hartwick & Schade (1990) conclude that this survey is probably the most efficient method of detecting faint quasars by slitless spectroscopy. Apart from being reputedly free of biases, this catalog is also interesting for our purposes as it covers a large range of QSO magnitudes and redshifts.

The galaxy catalogue of Odewahn & Aldering (1995), is also very appropriate for our purposes as it offers a photometrically homogeneous sample on a large region of the sky and contains colour and rough morphological information for the galaxies. As it was shown in Benítez & Martínez-González (1995) , selecting galaxies by their colors may produce dramatic effects in the behavior of the QSO-galaxy correlation function.

Section 2 shows the details of the data that made possible this work. Section 3 deals with QSO–galaxy cross-correlations, using two different approaches. The next section extracts the information on small angular distances using again two independent estimators. In the final section we discuss the implications and make an attempt at interpreting these results. It is concluded that bias effects in optically selected QSO catalogs may seriously affect the outcome of many studies based on this kind of quasars.

## 2. DATA

The statistical analysis done in this paper is based on two complete sets of galaxies and quasars respectively. The galaxy catalog compiled by Odewahn and Aldering (1995) was obtained from the APS scans of nine POSS-I photographic plates around the North

Galactic Pole, covering an area of  $289 \text{ deg}^2$ . In the calibration process, extra care was taken to avoid plate-to-plate variations in the photometry. The catalog lists  $O$  and  $E$  photographic magnitudes as well as a concentration index. This feature makes the sample especially interesting since it is possible to give a rough estimate of the morphology of each galaxy taking its  $O - E$  color and concentration. A suitable choice of subsamples allows an estimation of the distribution of galaxies on large scales. The region shows a conspicuous degree-sized structure including the Coma cluster as well as many filaments. The complete list comprises 36402 galaxies.

The quasar catalog comes from the CFHT/MMT survey (Crampton, Cowley & Hartwick 1989) based on gress spectra taken at the CFHT, using III-aJ photographic plates. Candidates were selected based on visual inspection searching for prominent emission lines (mainly C III, C IV, Mg II, and Ly- $\alpha$ ), or objects with unusually blue continua. A high percentage of confirmed candidates makes this survey one of the most efficient ones. There is supposed to be no evidence of significant incompleteness in the range  $0.3 < z < 3.4$  (Hartwick & Schade 1990). Crampton et al. surveyed several fields one of which — Field 1338+27 — is complete in a  $\sim 5.5 \text{ deg}^2$  region that overlaps with the galaxy sample and comprises 160 QSOs. Figure 1 shows these quasars superimposed on a smoothed image of the galaxy density. This region of the sky is quite remarkable because of a degree-sized filament of red galaxies stretching along the southern portion of the field. These two catalogs are thus ideal to explore effects of foreground structures on the number density and distribution of background QSOs.

In the next section we search for possible correlations both at small and large scales, using the complete sample of quasars and galaxies as well as several subsets listed below:

- Red ( $O - E \geq 1.5$ ) and Blue ( $O - E < 1.5$ ) Galaxies.
- Concentrated ( $c_{31} \geq 2.4$ ) and Non-Concentrated ( $c_{31} < 2.4$ ) Galaxies.

- Bright ( $B_j \leq 19.6$ ) and Faint ( $B_j > 19.6$ ) Quasars.
- High- $z$  ( $z > 1.6$ ) and Low- $z$  ( $z \leq 1.6$ ) Quasars.

The average values were the ones chosen to split the population of QSOs and galaxies so that each subsample has roughly the same number of objects. Just by eyeballing the subsamples we clearly see the large scale structure made of red and concentrated galaxies (i.e. old ellipticals). It is quite instructive to compare the results from Odewahn & Aldering (1995): The mean  $O - E$  color in the filamentary region is  $1.53 \pm 0.02$  compared to a bluer  $1.32 \pm 0.01$  in the interfilament area. Besides, the galaxies in the filaments are more concentrated, with a  $c_{31}$  index of  $2.226 \pm 0.006$  in contrast with  $2.153 \pm 0.005$  in the outside of the filaments.

### 3. QSO–Galaxy cross-correlations

This paper aims at estimating the cross-correlation between QSOs and galaxies. Such correlations give important information about background quasars being lensed by foreground galaxies. On the other hand, the QSO sample is assumed to be complete, which allows us to infer properties on their large scale distribution. The features of this sample make the analysis totally different from that of previous work. The galaxies are not weighed equally: the ones closer to the field center will be added more times when computing correlations than the ones in the outside. Hence, a comparison between the actual QSO catalog and a random QSO sample is essential to eliminate possible “contaminations” in the correlations coming from galaxy–galaxy clustering. In this section we explore the cross-correlation using two different methods: the correlation function, and a robust treatment comparing QSO and galaxy number densities.

### 3.1. Correlation Function

The correlation function gives the excess probability of finding a quasar–galaxy pair over that of a Poisson distribution. In this paper, care has to be taken in order to define a suitable correlation estimator. If the sample were big enough so that the density achieved homogeneity at large angular distances, then we could use the “standard” definition that normalizes the pair number with the area. Instead, we have to use a different normalization so that the function goes to zero at large angular distances. We count the number of galaxies inside rings centered at each quasar; then we compute an area correction by throwing 30000 random points homogeneously distributed over the QSO survey region, and then count the number of the random points which fall inside the rings. If  $n_g(\theta_i)$  represents the number of galaxies in a distance range  $\theta_i \leq \theta < \theta_i + \Delta\theta$  added for all the QSOs in the sample; and  $n_r(\theta_i)$  is the number of random points inside the same ring, then we define a normalized pair ratio:

$$\mathcal{N}_g(\theta_i) \equiv \frac{n_g(\theta_i)}{n_r(\theta_i)} \times \frac{\sum_i n_r(\theta_i)}{\sum_i n_g(\theta_i)} \quad (1)$$

A random distribution of points would give  $\mathcal{N}_g(\theta_i) = 1$ , hence its deviation from 1 is the correlation function. However, this function is extremely sensitive to the galaxy population, i.e. if there is a strong gradient in the number of galaxies across the field, then the correlation function will yield a bogus association caused by this gradient. The way out of this is computing the normalized pair ratio again for a simulated sample, which could be just a random distribution with the same number of QSOs as the original one. Instead, we decided to choose a slightly different comparison catalog: We divided the region in  $6 \times 6$  boxes ( $\sim 25' \times 22'$ ), shuffling and rotating them 100 times. This method will yield error bars that are more realistic than the ones obtained by throwing random points all over the QSO region. If we write the pair ratio for the random sample as  $\mathcal{N}_g^{\text{rnd}}(\theta_i)$  then the correlation

function is:

$$\omega_{\text{qg}}(\theta_i) = \frac{\mathcal{N}_g(\theta_i)}{\mathcal{N}_g^{\text{rnd}}(\theta_i)} - 1 \quad (2)$$

The correlation function is shown in Figure 2 for the complete sample as well as for partial subsamples taking red and blue galaxies and high and low redshift quasars. We used 200 simulated samples in each run to compute the correlation function and the error bars. The largest difference between subsamples appears when taking high and low redshift QSOs.

The  $z < 1.6$  population has a strong anticorrelation with galaxies that is more pronounced when restricting the foreground sample to red ( $O - E > 1.5$ ) galaxies, i.e. when dealing with the (elliptical) galaxies that dominate the gradient in number density. This paucity of quasars can be even checked by eye taking a look at Figure 1. To quantify this result, we applied a Spearman rank correlation test comparing the value of the correlation function against distance. We tried different cutoffs ( $\theta_{\text{cutoff}} = 120', 150', 180', 200'$ ) and all of them agreed with a confidence level  $\approx 100\%$  for the red-galaxy/low- $z$ -QSO subsamples. The bump at around  $80'$  is caused by the filament of galaxies in the low declination area. This bump gives a coarse estimate of the distance between the “centers of mass” of both quasar and galaxy catalogs.

### 3.2. QSO & Galaxy density diagrams

In order to double check this result, we used another method to estimate the negative cross-correlation found. The most straightforward way of checking whether there is a correlation on large scales is analysing the density of quasars as a function of galaxy density. We binned the region in  $n \times n$  boxes (in R.A. and Declination) counting QSOs and galaxies inside these boxes and sorting them with respect to the galaxy counts. Then we checked the results rebinning them as well as using different values of  $n$  to make sure the result was stable. A suitable value of  $n$  is given by the average quasar–quasar separation, roughly



10 arcminutes. Figure 3 shows several  $q = \rho_{\text{QSO}}/\langle\rho_{\text{QSO}}\rangle$  vs.  $\Delta\rho_g/\langle\rho_g\rangle$  plots for  $n = 15$  ( $10' \times 9'$  boxes), where the average galaxy density is computed in the region where the QSO survey is complete. The total  $15 \times 15$  points were rebinned for a better visualization. A Rank correlation applied to the complete sample of QSOs and galaxies with no rebinning gave a statistic which is  $1.7\sigma$  away from the expected value for a homogenous distribution, implying a confidence level of 95.5%.

However, the result is much more significant if we separate the list in high and low redshift quasars. Notice the remarkable difference between these two populations in the bottom panels of Figure 3. Taking the red-galaxies/high- $z$ -QSOs subsample we find the rank correlation statistic to lie  $2.6\sigma$  away from the prediction for a random sample, giving a confidence level of 99.5 %. Furthermore, we wanted to check for possible deviations from a Gaussian profile and performed the same treatment using 1000 random QSO catalogs. Only 4 out of 1000 gave a higher statistic than the real sample, thereby yielding a 99.6% confidence level, in agreement with the result obtained using the correlation function. Besides, the high redshift population behaves quite in the opposite way, suggesting only at a 92% confidence level a positive correlation with galaxy density. Anyway, the remarkable difference found between high and low redshift quasars is a clear hint that strong biasing might be present in one of the selection methods assumed to be most unbiased so far. This effect could be explained by the fact that high- $z$  quasars are detected searching for their prominent Lyman- $\alpha$  emission line whereas lower redshift candidates shift this line out of the spectral range of the photographic emulsion and so their detection is based on weaker — thereby harder to detect — emission lines such as C IV, Mg II and so on.

#### 4. Sub-degree QSO-Galaxy cross-correlations

In order to check for the existence of QSO-Individual galaxy gravitational lensing effects on the sample, we have to extract information on sub-degree angular distances, taking away the contribution from the large scale galaxy distribution. We have used two independent methods that agree in finding no correlation at these distances with a  $\approx 100\%$  confidence level. We have excluded quasars with redshift  $z < 0.4$  to make sure the only possible correlations on these short angular distances come from line-of-sight effects such as gravitational lensing or dust absorption. A final list with 146 QSOs is thus considered.

##### 4.1. Nearest Neighbor Estimation

Small scale correlations are harder to check in this work: The list does not have many quasars which means poor statistics. Besides, there is only one region over which the quasar catalog is defined. Hence we will be adding the same galaxies many times when computing correlations. Gradients in the galaxy density might yield bogus QSO-galaxy correlations. It is thus necessary to cross-check the actual sample with random QSO catalogs which are roughly distributed the same way on large scales as the original survey. Following previous work on the subject (Thomas, Webster & Drinkwater 1995), we chose the closest QSO-galaxy distance as the estimator of correlations on short scales. A histogram with these distances for each quasar can be readily compared with the same one for a random sample made by shifting 10 arcminutes the original survey four times: North, South, East and West. A rank correlation test comparing these two distributions finds no difference between them with a  $\approx 100\%$  confidence level (see Figure 4). We applied the same treatment to the bright ( $B_j < 19.6$ ) and faint ( $B_j > 19.6$ ) QSO subsamples. Gravitational lensing predicts these two catalogs should have different short-scale correlations because of the different slope in the number counts function. No correlation was found for either case

at a  $\approx 100\%$  confidence level.

To double check this result, we considered a list comprising all QSOs in the galaxy region ( $289 \text{ deg}^2$ ) from the catalog of Veron-Cetty & Veron (1996). A total of 508 QSOs were used and no excess or defect was found again at  $\approx 100\%$  confidence. Even though this catalog is not complete, we do not anticipate a strong difference from a complete sample since this estimator only has to do with short angular distances and Veron’s list compiles many different catalogs which should not imply a bias on such scales. It would be very interesting to follow the same steps only for radio-loud quasars as previous studies found evidence for positive correlations explained by a double magnification bias effect. However, it is not possible to do this because the list has only 30 radio-loud quasars inside this area, too few to infer a statistically significant value.

## 4.2. Sub-degree Correlation Estimator

Another way of checking for possible cross-correlations on small angular distances is defining a correlation estimator that is similar to the correlation function used above. The actual sample, though, is compared with a random sample of QSOs that preserve the large scale structure of the original catalog. It is obtained by dividing the region in  $6 \times 6$  boxes ( $\sim 25' \times 22'$ ) and randomly throwing inside each box the same number of QSOs as the ones found in the original sample. The correlation estimator is computed the same way as the correlation function defined above (equations 1 and 2). The result is shown in Figure 5 using again 200 simulated catalogs: The error bars are compatible with no cross-correlation for either the whole catalog or partial subsamples, in agreement with the outcome from the nearest neighbor histogram.

## 5. DISCUSSION

The effect of gravitational lensing on short angular distances on this subsample of the optically-selected CFHT/MMT survey can be seen in Figure 4. There is no significant correlation on angular distances  $\theta \lesssim 10'$ . This result was further checked with all the quasars listed in the Veron-Cetty & Veron (1996) catalog, for which no correlation was found either. The overdensity factor  $q$  depends on the lens magnification  $\mu$  and the slope of the QSO number counts function  $\alpha$  as:

$$q \propto \mu^{\alpha-1} \quad (3)$$

The slope of the number counts function determines the correlation: positive for a steep slope ( $\alpha > 1$ ) and negative for a shallow slope ( $\alpha < 1$ ). Figure 6 shows the number counts function versus  $B_j$  photographic magnitude and redshift for the complete CFHT sample (top panels) and separated in two populations lying on the high and low galaxy density areas respectively (bottom panels). The complete sample has two slopes: The bright section has  $\alpha = 1.7$  (positive correlation expected) whereas the faint section has  $\alpha = 0.7$  (slightly negative correlation). Separating the samples at the point where the slope changes (around  $B_j \sim 19.5$ ) allows us to estimate whether the lensing effect is noticeable. No correlation was found, with a  $\approx 100\%$  confidence level using a Spearman rank correlation test.

The new issue that arises in this work has to do with possible biasing in optically selected samples. We were lucky to use the best QSO survey that minimizes selection effects (line emission). Yet, we found a very significant QSO–galaxy anticorrelation on large scales that — on a first estimate — could be associated with dust tracing the distribution of foreground galaxies. Figures 2 and 3 show this negative correlation that can be even checked by eye in Figure 1. However, when we take into account different subsamples, we find the low redshift population is the one responsible for this negative correlation, as can be shown in the bottom panels of Figure 2. Selecting a subsample of low- $z$  quasars and red galaxies

(which trace most of the galaxy density gradient), we find a significant anticorrelation with a  $> 99.6\%$  confidence level. Taking a look at the density–density diagrams, a defect of 50% in the low- $z$ -QSO number counts appears in high galaxy density regions. An estimation for the complete sample obtained either from the linear fits from Figure 3 or by integrating the correlation function in Figure 2 yields a quasar defect around 10%. That is, if the quasar list were truly complete, we would expect around 15 extra QSOs, most of them with redshifts  $z < 1.6$ .

This effect can only be explained by a redshift–dependent bias on the selection process. This has been already considered by Hartwick & Schade (1990) but so far it had not been possible to quantify. High redshift quasars are mainly detected through their strong Lyman- $\alpha$  emission peak, whereas low redshift objects must be detected using less prominent emission lines that may introduce this bias. If we assume a spectral sensitivity range of  $\lambda\lambda$  3500–5400 Å for the CFHT survey with III-aJ plates (Crampton, Schade & Cowley 1985), we get a redshift range  $1.9 < z < 3.4$  using the Lyman- $\alpha$  peak;  $1.3 < z < 2.5$  using C IV; and  $0.25 < z < 0.9$  using Mg II. If we take into account the stronger flux emitted in the Lyman- $\alpha$  peak, we may expect to find more QSOs at high redshift using this selection process. Hence, as opposed to earlier claims, the blue gress surveys could be significantly susceptible to the “Ly- $\alpha$  window bias” altering the completeness level of the catalogs obtained this way. Besides, we have found that the selection method is less efficient around high galaxy density areas, where the crowded gress plates might make searching for faint emission lines quite a challenge. This result should be explored further using QSO surveys on regions with a strong galaxy density gradient (that is essential because the anticorrelation obtained in this work is only significant taking the red galaxies that trace the large scale structure). The important implications that this correlation has on the completeness level of surveys as well as on any study dealing with quasar–galaxy associations make the effort worthwhile.

We are happy to thank S.C.Odewahn & G.Aldering for sending us a machine-readable file of their galaxy catalog. I.F. and N.B. acknowledge a Ph.D. scholarship from the ‘Gobierno de Cantabria’, and the Spanish MEC, respectively. I.F.,N.B. and E,M.-G. acknowledge financial support from the Spanish DGES under contract PB95-0041.

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Fig. 1.— Region where the QSO catalog used is complete ( $\approx 5.5\text{deg}^2$ ). North and East are up and left respectively. The center corresponds to R.A.= $13^h37^m13.25^s$  ; Dec.=  $+27^\circ20'00''$  (B1950.0). The 160 quasars are shown as stars and the galaxies are smoothed by convolution with a gaussian (FWHM $\approx 1'$ , roughly the average galaxy–galaxy separation) into a density function shown with a grey scale. Notice the paucity of quasars around the high density filament in the bottom part of the figure.

Fig. 2.— Cross–Correlation Function.  $1-\sigma$  error bars are obtained by comparing the real QSO sample with 200 random catalogs with the same number of quasars. The subsample comprising red galaxies and low redshift quasars gives a non–zero cross–correlation function suggesting a bias in grism–selected surveys against detection of low redshift quasars around high galaxy density. A Spearman Rank correlation test gives a  $\approx 100\%$  confidence level. This bias can be due to the need to detect fainter emission lines in contrast with high- $z$  QSOs for which the strong Lyman- $\alpha$  peak falls within the spectral range of the detector.

Fig. 3.— Density–density plots for different samples: The galaxy density gradient ( $\Delta\rho/\langle\rho\rangle$ ) is plotted versus QSO number ( $q=N_q/\langle N_q\rangle$ ). Top panel: Complete sample of QSOs and galaxies. The bottom panels show the same plot for a few subsamples: red( $O - E > 1.5$ ) and blue galaxies and high and low redshift QSOs. The line gives a least squares fit to each diagram. Notice the agreement of this method with the cross–correlation function shown in figure 2: the subsample with red galaxies (which trace the gradient in the galaxy density) and low–redshift quasars is the one with a significant negative cross–correlation. The remaining panels give a result compatible within error bars with no correlation whatsoever.

Fig. 4.— Nearest neighbor estimator. The top panel shows the result for the QSO catalog used in this work. The bottom panel serves as a check using all quasars found in the list from Veron-Cetty & Veron (1996) lying inside the  $289\text{deg}^2$  galaxy region. Thick lines represent the actual sample whereas thin lines show the random catalogs. The result is compatible



with no cross-correlation on sub-degree angular distances (confidence level  $\approx 100\%$  using a rank order test).

Fig. 5.— Correlation estimator for small angular distances using 200 random catalogs to match against the real sample.  $1\text{-}\sigma$  error bars are shown. the top panel shows the result for the complete catalogs. Bottom panels show the function for different subsamples. This method is totally independent from a nearest neighbor estimator (fig.4), yet it agrees in finding no cross-correlation for any subsample.

Fig. 6.— QSO number counts versus magnitude and redshift. The top panels show the functions for the complete sample; the bottom panels take the high (triangles) and low (squares) galaxy density regions separately. Poisson error bars are shown. The slopes of the linear fit in the number counts versus magnitude diagram are 1.7 and 0.7 for  $B_j \leq 19.5$  and  $B_j > 19.5$  respectively.











